Problem Set 8. Due 11/13

Reminder: You must acknowledge your sources and collaborators (even if it is "none", you must write so). Failure to do so on this problem set will result in an automatic 2-point deduction.

1. Prove that a connected graph G is k-edge-connected if and only if each block of G is k-edge-connected.

2. Prove that a graph G is 2-connected if and only if for any three vertices x, y, z there is a path from x to z containing y.

3. Let G be a 3-connected graph. Let x, y, and z be three vertices in G. Prove that there is a cycle containing x, y, and z.

4. Let G be a 3-connected graph with at least 6 vertices. Prove that G contains a cycle of length at least 6.

5. Let G be a graph with an Eulerian tour. Prove or disprove each of the following:

   (a) If G has an even number of vertices, then it has an even number of edges.

   (b) For edges e and f sharing a vertex, G has an Eulerian tour in which e and f appear consecutively.

6. Show that if $k > 0$ then the edge set of any connected graph with $2k$ vertices of odd degree can be split into k trails.

7. An oriented complete graph (i.e. one that has exactly one directed edge between any pair of vertices) is called a *tournament*. Show that every tournament contains a (directed) Hamilton path (i.e., a path containing all vertices of the graph such that all edges on the path are directed the same way).

8. Let G be a 3-connected graph with no independent set of size 4. Prove that G has a Hamiltonian cycle.