# Practice Midterm 3

Time: 80 minutes.

5 problems worth 10 points each.

No electronic devices. You may bring **two sheets of notes** on letter-sized paper (total four sides front and back) **in your own handwriting**. Typed, printed, or photocopied notes are **forbidden**.

You must provide justification in your solutions (not just answers). You may quote theorems and facts proved in class, course textbook/notes, or homework, provided that you state the facts that you are using.

1. Determine whether each of the following statement is TRUE or FALSE, and provide a short justification or a counterexample (a correct answer without justification receives zero credit).

   (a) If $G$ is a connected planar graph, then any planar embedding of $G$ always has the same number of faces.

   (b) If $G$ is a connected $d$-regular graph with $d \geq 1$, then its line graph $L(G)$ contains an Eulerian tour.

2. Does there exist a connected graph with a cut vertex whose edge set can be partitioned into perfect matchings?

3. Let $G$ be a bipartite graph with $n$ vertices on both sides and minimum degree at least $n/2$. Prove that $G$ has a perfect matching.

4. Let $k \geq 1$. Let $G$ be a $2k$-edge-connected graph. Let $s_1, \dots, s_k, t_1, \dots, t_k$ be distinct vertices. Show that there are edge disjoint paths $P_1, \dots, P_k$ such that each $P_i$ starts at $s_i$ and ends at $t_i$.

5. Prove that the union of $k$ planar graphs is $6k$-colorable.